ALGEBRA.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS TO PROBLEMS.

3. Proposed by Professor H. A. WOOD, A. M., Hoboken, New Jersey.

If $x^6-y^6=665$, and $x^3y+xy^3=78$, find x and y.

Solution by Professor G. B. M. ZERR, A. M., Principal of Schools, Staunton, Virginia.

Let $x^2-y^2=s$, $xy=p$. Then $x^6-y^6=s^3+3p^2s-665$, $\therefore p^2=\frac{665-s^3}{2s}$....(1). Also, $p\sqrt{s^2+4p^2}=78$, or $p^2s^2+4p^4=6084$....(2).

(1) in (2) gives, $s^6-3325\,s^3-54756s^2+1768900=0$.

$\therefore$ $(s-5)(s^5+5s^4+25s^3-3200s^3-70756s^3-353780)=0$.

$\therefore s=5$. $\therefore x^2-y^2=5$, $xy=6$, and now $x=\pm3$, $y=\pm2$.

Solved also by *P. S. Berg*, and Professors *Scheffer* and *Whitaker*.

4. Proposed by L. E. PRATT, Tecumseh, Nebraska.

If Σm, Σm^3, Σm^5, ... Σm^{2n-1} are the sums of the 1st, 3rd, 5th, ... $(2n=1)$th powers of the first m natural numbers, prove that $n\Sigma m^{2n-1}+\frac{n(n-1)(n-2)}{|\underline{3}}\Sigma m^{2n-3}+\frac{n(n-1)(n-2)(n-3)(n-4)}{|\underline{5}}\Sigma_m{}^{2n-5}+......=2^{n-1}\Sigma^n_m$.

Solution by the Proposer.

Assume the natural series 1, 2, 3, 4, 5, ... $(m-1)^n$, ... m^n, ... $(m+1)^n$ ....(1). That portion of (1) included between $(m-1)^n$ and $(m+1)^n$ may be written

$$\frac{\{(m^n-1)-(m-1)\}\{(m-1)^n+m^n\}}{2},\ m_n,\ \frac{\{(m+1)^n-(m^n+1)\}\{m^n+(m+1)^n\}}{2}\text{....(2)}.$$

Collecting all the terms having the form m^n contained in (2), we have $\frac{m^n\{(m+1)^n-(m-1)^n\}}{2}$....(3).

This expression is the general term of a series, involving the first m terms of (1), whose sum, as may be seen by changing m into $m-1$, $m-2$, ..., is $\frac{m^n(m+1)^n}{2}$, or $2^{n-1}\Sigma^n_m$. If we now expand (3), we have

$$n\,m^{2n-1}+\frac{n(n-1)(n-2)}{|\underline{3}}m^{2n-3}+\frac{n(n-1)(n-2)(n-3)(n-4)}{|\underline{5}}m^{2n-5}+\text{....(4)}.$$

Each term of (4) may be regarded as a type-term of a series, where m may have the same values as in (3).

$$\therefore\ n\,\Sigma m^{2n-1}+\frac{n(n-1)(n-2)}{|\underline{3}}\Sigma m^{2n-3}+....=2^{n-1}\Sigma^n_m$$

[By Σ^n_m is meant $(\Sigma_m)^n$. The latter is the preferable way of writing it. Several contributors misunderstood Σ^n_m as given by the proposer.—Editor.]

5. Proposed by WILLIAM E. MAY, 500 Union St., Knoxville, Tennessee.

A, B, C went to market, each with 10, 30, and 50 eggs, respectively. On their